

The (ring-theoretical) *center* of an algebra in a semidegenerate variety was introduced by Vaggione in [?], generalizing elements of the form $(0, 1)$ in direct products $\mathbf{A} \times \mathbf{B}$ (as in the case of central idempotent elements in rings with identity and neutral complemented elements in a bounded lattice). Under some structural hypotheses (like *Boolean factor congruences*), there exists a formula $\omega(z, w, x, y)$ which defines the kernel of the projection $\pi_1 : \mathbf{A} \times \mathbf{B} \rightarrow \mathbf{A}$ by using the associated central element $(0, 1)$ and its complement $(1, 0)$ as parameters. In general, one must take tuples of terms $\vec{0} = 0_1, \dots, 0_N$ and $\vec{1}$ similarly, and central elements are of the form $((0_1, 1_1), \dots, (0_N, 1_N)) \in (A \times B)^N$.

The authors study the definability of the center and the relation “ e and f are complementary central elements” under the hypothesis that there exists an existential formula ω as above, focusing in the case of universal and equational definability. One the main results (presented for the case $N = 1$ in this review) states that for every \mathcal{V} under the hypothesis, the following are equivalent:

- The center is definable by a set of universal sentences;
- there exists a ternary (“*decomposition*”) term $u(z, x, y)$ such that

$$\mathcal{V} \models u(0, x, y) = x = u(1, y, x);$$

- subalgebras of products $\mathbf{S} \leq \mathbf{A} \times \mathbf{B}$ with $(0, 1) \in \mathbf{S}$ factorize.

Moreover, the center is definable by a single universal sentence if and only if the class \mathcal{V}_{DI} of directly indecomposables of \mathcal{V} is elementary. In the same line, it is shown that for every locally finite variety \mathcal{V} in which the center is definable by a set of equations, \mathcal{V}_{DI} is a universal class.

Similar results are obtained considering universal definability of complementarity, by using a quaternary decomposition term U :

$$\mathcal{V} \models U(0, 1, x, y) = x = U(1, 0, y, x).$$

The authors also provide definitions of the operations and the order of the Boolean algebra of central elements in varieties with decomposition terms. The operations are given by terms in the case of a ternary decomposition.