

The authors refine several results in the recent literature on constraint satisfaction problems (CSP). In doing so, they identified a basic construction in universal algebra, *reflection*, that turns out to be very useful in deciding whether  $\text{CSP}(\mathbb{B})$  reduces to  $\text{CSP}(\mathbb{A})$  for countable  $\omega$ -categorical relational structures  $\mathbb{A}$  and  $\mathbb{B}$ . Here, a structure  $\mathbb{A}$  is  *$\omega$ -categorical* if the component-wise action of its automorphism group on  $A^n$  has only finitely many orbits. In this review, all relational structures are assumed to be at most countable and  $\omega$ -categorical.

Three constructions on relational structures are relevant for the complexity of the CSP: interpretations by pp-formulas, homomorphically equivalent (He) structures, and adding singleton unary relations to an  $\omega$ -categorical *core* (i.e., a structure in which automorphisms are dense in the set of endomorphisms, in the topology of pointwise convergence). We say that  $\mathbb{B}$  is *pp-constructed* from  $\mathbb{A}$  if it can be obtained from the latter by a finite application of these constructions; in this case  $\text{CSP}(\mathbb{B})$  is log-space reducible to  $\text{CSP}(\mathbb{A})$ . The authors single out a particular case of interpretation, *pp-powers* (Ppp) and show that  $\mathbb{B}$  can be pp-constructed from  $\mathbb{A}$  if and only if  $\mathbb{B} \in \text{He}(\text{Ppp}(\mathbb{A}))$ .

Reflections generalize retractions of structures: Given an algebra  $\mathbf{A}$ , a set  $B$  and mappings  $B \rightarrow A$  and  $A \rightarrow B$ , an algebra with universe  $B$  (in the same signature as  $\mathbf{A}$ ) can be defined by “translating” of the operations of  $\mathbf{A}$  along the previous mappings. Several (in)equalities among compositions of the operators  $\text{H}, \text{S}, \text{P}$  and  $\text{R}$  (reflection) are shown; for instance, it is proved that  $\text{RP}(\mathcal{K})$  is equal to the closure of  $\mathcal{K}$  under  $\text{R}, \text{H}, \text{S}, \text{P}$  for a class of algebras  $\mathcal{K}$ . These operators apply as well to function clones, and characterizations of three aforementioned basic constructions on relational structures are obtained in terms of reflection and the others operators. We single out the following equivalences:

- $\mathbb{B} \in \text{He}(\mathbb{C})$  for some  $\mathbb{C}$  pp-definable from  $\mathbb{A}$  if and only if  $\text{Pol}(\mathbb{B}) \in \text{ER}(\text{Pol}(\mathbb{A}))$ ;
- $\mathbb{B}$  can be pp-constructed from  $\mathbb{A}$  if  $\text{Pol}(\mathbb{B}) \in \text{ERP}_{\text{fin}}(\text{Pol}(\mathbb{A}))$ ,

where  $\text{E}$  is the expansion operator. The condition  $\text{Pol}(\mathbb{B}) \in \text{ERP}_{\text{fin}}(\text{Pol}(\mathbb{A}))$  is then analyzed in terms of continuous clone mappings preserving compositions with invertibles.

It is also noteworthy the characterization of varieties defined using height 1 (resp., linear) equations, as those classes of algebras closed under  $\text{R}$  (resp., retractions) and  $\text{P}$ . In these proofs, the notion of “*h1*” clone homomorphism (i.e., preserving identities of height 1) plays a role. It also appears in the section on colorings of clones by relational structures, where succinct characterizations of congruence modular and  $n$ -permutable (for some  $n$ ) varieties are given. This has applications to the problem of primality of the corresponding Maltsev filters in the lattice of interpretability types of varieties.